Appendix 3: Formal Definition of SEM

There are two versions of SEM: a single-trial version and a multiple-trial version. The single-trial version does not model intertrial effects: The tokens in short-term memory are restricted to those from the most recent list. The multiple-trial version is more general, including tokens from previous trials, together with general context, phonological decay and rehearsal. These versions are formalised below.

Note that the formalism can obscure the relatively simple mechanisms underlying the model, which are described verbally in Chapters 5 and 6. In particular, some of the parameters and variables introduced in Chapter 5 require additional suffices for clarification below. In general, parameters are in upper-case and variables are in lower-case. The large number of parameters reflects the generality of the model. In most cases, these parameters are either fixed, constrained by the experimental design, or constrained by values of other parameters.

Single-trial Version of SEM

The single-trial version takes a single list of items, and simulates N_L independent trials at serial recall of that list. Each trial can be split into two stages of presentation and recall.

Presentation

A token is created for each item at position $p=1..N_P$ of the list. Specifically, for each group $g=1..N_G$ and each item in group g, $i=1..N_I(g)$, a token t is created with positional codes $\boldsymbol{p}_I^{(t)}$ and $\boldsymbol{p}_G^{(t)}$.[1]

The vector $\boldsymbol{p}_I^{(t)} = < x_I(i) \;\; y_I(i) >$ is a positional code for the position of item i within group g, where $x_I(i)$ and $y_I(i)$ are the strengths of markers for the start and end of that group:

$$x_I(i) = S_{0,I} S^{i-1} \qquad y_I(i) = E_{0,I} E_I^{N_I - i} \qquad \text{Equation 10-1}$$

1. Ungrouped lists can be modelled either with $N_G=1$ and $N_I(1)=N_P$, or with $N_G=0$, in which case there are no group start and end markers and $i=1..N_P$.

where $S_{0,I}$ and S_I, are parameters reflecting the initial strength of the start marker and the rate of change of its strength, and $E_{0,I}$ and E_I are parameters reflecting the initial strength of the end marker and the rate of change of its strength.

Associated with each positional code $\boldsymbol{p}_I^{(t)}$ is a quantity $d_I^{(t)}$, reflecting the noise in the encoding of that position. The value of $d_I^{(t)}$ is drawn from a zero-mean Gaussian distribution with standard deviation D_I for each position p.

The vector $\boldsymbol{p}_G^{(t)} = < x_G(g)\ y_G(g) >$ is a positional code for the position of group g in the list, where $x_G(g)$ and $y_G(g)$ are the strengths of markers for the start and end of that list:

$$x_G(g) = S_{0,G} S_G^{g-1} \qquad y_G(g) = E_{0,G} E_G^{N_G - g}$$ Equation 10-2

where $S_{0,G}$ and S_G are parameters reflecting the initial strength of the start marker and the rate of change of its strength, and $E_{0,G}$ and E_G are parameters reflecting the initial strength of the end marker and the rate of change of its strength.

Associated with each positional code $\boldsymbol{p}_G^{(t)}$ is a quantity $d_G^{(t)}$, reflecting the noise in the encoding of that position. The value of $d_G^{(t)}$ is drawn from a zero-mean Gaussian distribution with standard deviation D_G for each group g.

Recall

For each response $r=1..N_P$, a cue is generated with positional codes $\boldsymbol{p}_I^{(r)}$ and $\boldsymbol{p}_G^{(r)}$, as defined in Equation 10-1 and Equation 10-2. The noise associated with reinstating these positional codes is given by $d_I^{(r)}$ and $d_G^{(r)}$, again drawn from zero-mean Gaussian distributions with standard deviations D_I and D_G respectively. The variable $d_I^{(r)}$ is drawn for each response; the variable $d_G^{(r)}$ is drawn for each new group recalled.

The retrieval of an item as response r can be divided into six stages:

Stage 1: Cuing

The positional codes $\boldsymbol{p}_I^{(r)}$ and $\boldsymbol{p}_G^{(r)}$are matched against the positional codes, $\boldsymbol{p}_I^{(t)}$ and $\boldsymbol{p}_G^{(t)}$, of the $t=1..N_T$ tokens in short-term memory[2], cuing each with strength $q^{(t)}(r)$:

2. (N_T=N_P in the single-trial version, but not the multiple-trial version)

$$q^{(t)}(r) = \bar{m}(\boldsymbol{p}_I^{(t)}, \boldsymbol{p}_I^{(r)}, d_I, M_I)\,\bar{m}(\boldsymbol{p}_G^{(t)}, \boldsymbol{p}_G^{(r)}, d_G, M_G) \qquad \text{Equation 10-3}$$

where $d_I = d_I^{(t)} + d_I^{(r)}$, $d_G = d_G^{(t)} + d_G^{(r)}$, and the parameters M_I and M_G are match criteria, representing the degree to which positional codes must match in order to be cued, as defined by the linear thresholding function, $\bar{m}$:

$$\bar{m}(\boldsymbol{p}, \boldsymbol{q}, d, M) = \begin{cases} 0 & m(\boldsymbol{p}, \boldsymbol{q}, d) < M \\ m(\boldsymbol{p}, \boldsymbol{q}, d) & m(\boldsymbol{p}, \boldsymbol{q}, d) \geq M \end{cases} \qquad \text{Equation 10-4}$$

where m is the noisy match between positional codes:

$$m(\boldsymbol{p}, \boldsymbol{q}, d) = o(\boldsymbol{p}, \boldsymbol{q}) + d \qquad \text{Equation 10-5}$$

and $o(\boldsymbol{p},\boldsymbol{q})$ is the overlap between position codes $\boldsymbol{p}$ and $\boldsymbol{q}$:

$$o(\boldsymbol{p}, \boldsymbol{q}) = \sqrt{\boldsymbol{p} \cdot \boldsymbol{q}} \times exp\left(-\sqrt{\sum_k (p_k - q_k)^2}\right) \qquad \text{Equation 10-6}$$

where the summand k is over the (two) components of vectors $\boldsymbol{p}$ and $\boldsymbol{q}$.

Note that the positional uncertainty functions, $f(i,j)$, representing the overlap between all $i,j=1..N_p$ positions of a sequence, are given by:

$$f(i, j) = o(\boldsymbol{p}_I^{(i)}, \boldsymbol{p}_I^{(j)})\,o(\boldsymbol{p}_G^{(i)}, \boldsymbol{p}_G^{(j)}) \qquad \text{Equation 10-7}$$

These functions are also the average, unthresholded cued strength of items at each position i during recall of each response j.

Stage 2: Categorical Selection

Items compete for selection with a strength proportional to their most strongly cued token. Specifically, the categorical (type) representations of all items $u=1..N_V$ in the vocabulary compete with strength $c_C^{(u)}$, where:

$$c_C^{(u)} = max\{q^{(t)}|_{i(t)=u}\}(1 - s_C^{(u)}) + n_C \qquad \text{Equation 10-8}$$

where *i(t)* is the identity of (the item corresponding to) token *t*, $s_C^{(u)}$ is the suppression of the categorical representation of item *u*, and n_C is random noise drawn from a zero-mean Gaussian distribution with standard deviation G_C for each item *u*. The strongest item *u** is selected and passed to Stage 3.

Note that the "max" function in Equation 10-8 could be changed for another function, such as the "sum" of cued strengths of all tokens of a particular item. This choice really depends on empirical data concerning the effect of repeated items in recall (Chapter 7).

Stage 3. Suppression

The categorical representation of the item selected at Stage 2 is suppressed, such that $s_C^{(u*)}=1$. Meanwhile, the suppression of all other items *u*, except *u**, wears off according to the update rule:

$$s_C^{(u)} \rightarrow s_C^{(u)} exp(-R_S) \qquad \text{Equation 10-9}$$

where R_S is the rate of decay of suppression. This decay is assumed to operate in real-time, though for convenience, suppression is only updated during each response and is assumed to have worn off completely between trials.

Stage 4. Phonological Retrieval

The item *u** selected from Stage 2 is matched against a second set of phonological representations in order to articulate a response. The possibility of phonological confusions arises at this stage. Specifically, competition is held over a set of phonological item representations v=$1..N_V$, each of which competes with strength, $c_P^{(v)}$:

$$c_P^{(v)} = c_C^{(v)} + p(v, u^*)\, a_P^{(v)}\, (1 - s_P^{(v)}) + n_P \qquad \text{Equation 10-10}$$

where *p(v,u)* is the phonological similarity between items *v* and *u*, $a_P^{(v)}$ is the activation of the phonological representation of item *v*, $s_P^{(v)}$ is the suppression of the phonological representation of item v, and n_P is random noise drawn from a zero-mean Gaussian distribution with standard deviation G_P for each item v.

The value of $p(v,u)$ is such that $p(v,u)=1$ if $v=u$, $p(v,u)=P_S$ if item v and item u are phonologically similar (i.e. confusable), and $p(v,u)=P_D$ if they are dissimilar (i.e., if one is nonconfusable). The value of $a_P^{(v)}$ is such that $a_P^{(v)}=A_P$ if item v was in the most recent list, and $a_P^{(v)}=0$ otherwise. The strongest item v^* is passed on to Stage 5.

Stage 5. Thresholding and Guessing

If the strength of the item retrieved from Stage 4 is above a guessing threshold T_G, such that $c_P^{(v^*)}>T_G$, it is passed directly to Stage 6.

If the strength of the item selected from Stage 4 is below the guessing threshold, but above an omission threshold T_O, such that $T_O<c_P^{(v^*)}<T_G$, then an item is guessed instead. This guessing is over the $v=1..N_V$ phonological representations, which compete with strengths $c_G^{(v)}$, given by:

$$c_G^{(v)} = a_P^{(v)} (1 - s_P^{(v)}) + n_G \qquad \text{Equation 10-11}$$

where n_G is a random noise drawn from a zero-mean Gaussian distribution with standard deviation G_G for each item v. The item winning this competition is passed to Stage 6.

If the strength of the item retrieved from Stage 4 is below the omission threshold, such that $c_P^{(v^*)}<T_O$, then no item is recalled and an omission is indicated instead. The next response is then cued (returning to Stage 1).

Stage 6. Output

The item v^* selected or guessed after Stage 4 is output as response r. Its phonological representation is suppressed, such that $s_P^{(v^*)}=0$, and the suppression of other phonological representations decays in the same manner as Equation 10-9. Note that the value of $s_P^{(v)}$ is independent of the value of $s_C^{(u)}$ (i.e., the categorical and phonological representations of the same item represent distinct loci of suppression).

The above process then repeats for response $r+1$, returning to Stage 1.

Multiple-trial Version

In this version of SEM, short-term memory is assumed to contain tokens from previous trials as well as the most recent trial (i.e., $N_T > N_P$). These tokens include a new component which represents general (nonpositional) context, which cannot be reinstated at recall. In addition, each item recalled is recoded as a new token (coded with its recall position, irrespective of whether that is correct), a process which also reactivates its phonological representation. Finally, the activation of phonological representations is assume to decay over time, to reflect transient nature of phonological information in short-term memory.

The multiple-trial version takes N_L different lists and recalls each one once. Recall of each list $l=1..N_L$, with positions $p=1..N_P(l)$, can be split into presentation, retention, recall and intertrial intervals. Only the differences between the multiple-trial version and the single-trial version are formalised below.

Presentation

Each token t has three components $\boldsymbol{p}_I^{(t)}$, $\boldsymbol{p}_G^{(t)}$and $\boldsymbol{p}_C^{(t)}$, where $\boldsymbol{p}_I^{(t)}$, $\boldsymbol{p}_G^{(t)}$ are the positional contexts defined in Equation 10-1 and Equation 10-2, and $\boldsymbol{p}_C^{(t)}$ is a one-dimensional vector representing the general (nonpositional) context when token t was created. For mathematical convenience, the current general context is represented by the constant value $E_{0,C}$, and the general context of all tokens in memory is updated each time the general context changes. Thus, each time an item is presented, its token is created with $\boldsymbol{p}_C^{(t)} = \langle E_{0,C} \rangle$. During subsequent contextual changes (e.g., presentation of other items), the general context of all tokens is updated according to:

$$\boldsymbol{p}_C^{(t)} \rightarrow \boldsymbol{p}_C^{(t)} E^c{}_{0,C} \qquad \text{Equation 10-12}$$

where E_C represents the rate of contextual change, and c represents the number of contextual changes (episodes). During presentation of each item, c is parameterised by $c=C_P$.

Presentation of an item v also activates its phonological representation by an amount $a_P^{(v)}=A_P$, while the activation of other phonological representations decays as follows:

$$a_P^{(v)} \rightarrow a_P^{(v)} exp(-cR_P) \qquad \text{Equation 10-13}$$

where R_P is the rate of decay of phonological activations, and c is the number of episodes (again, $c=C_P$ during presentation). All items have a baseline activation of $a_P=0$ at the start of the first trial.

Note that when the list length is unpredictable from trial to trial (as in Experiment 5), the behaviour of end markers during presentation differs to that in Equation 10-1. The strength of the end marker coding position in group (or position in list for ungrouped lists) becomes a function of the minimum expected list length, N_M, such that:

$$y_I(i) = \begin{matrix} E_{0,I}E_I^{N_M-i} & i < N_M \\ E_{0,I} & i \geq N_M \end{matrix} \qquad \text{Equation 10-14}$$

During recall, when the length is known, the end marker behaves as before (Equation 10-1).

Retention Interval

During the retention interval, the general context of all tokens is updated according to Equation 10-12, and the phonological activations of items decay according to Equation 10-13, where $c=C_D$ represents the number of episodes during the (filled) delay before recall.

Recall

For each response $r=1..N_P(l)$, a cue is generated with positional context $\boldsymbol{p}_I^{(r)}$, $\boldsymbol{p}_G^{(r)}$ and general context $\boldsymbol{p}_C^{(r)}$, where $\boldsymbol{p}_C^{(r)}$ is always the current context $< E_{0,C} >$. The noise associated with reinstating the positional codes is given by $d_I^{(r)}$ and $d_G^{(r)}$, as before.

The multiple-trial and single-trial versions differ in Stages 1 and 5 of recall:

Stage 1: Cuing

The positional and general context of the cue is matched against that of the $t=1..N_T$ tokens in short-term memory, cuing each with strength $q^{(t)}(r)$:

$$q^{(t)}(r) = \bar{m}(\boldsymbol{p}_I^{(t)}, \boldsymbol{p}_I^{(r)}, d_I, M_I)\,\bar{m}(\boldsymbol{p}_G^{(t)}, \boldsymbol{p}_G^{(r)}, d_G, M_G)\,o(\boldsymbol{p}_C^{(t)}, \boldsymbol{p}_C^{(r)}) \qquad \text{Equation 10-15}$$

where $\overline{m}$ is the thresholded overlap as in Equation 10-4. (For simplicity, no noise or match criterion is assumed for the general context.)

Stage 6: Output

The item v^* selected after thresholding in Stage 5 is output as response r, as before. In addition however, its phonological representation is reactivated, such that $a_P^{(v^*)}=A_P$, and it is recoded as a new token in short term memory, with positional and general context given by $\boldsymbol{p}_I^{(r)}$, $\boldsymbol{p}_G^{(r)}$ and $\boldsymbol{p}_C^{(r)}$ (i.e., that of the cue for response r), together with the noise associated with the encoding process, as before.

Finally, the general context of all tokens is updated according to Equation 10-12, and the phonological activations of items decay according to Equation 10-13, where $c=C_R$ represents the number of episodes during the recall of each item.

Note that, in order to handle uncertain responses in Experiments 4 and 5, the multiple-trial version also includes an uncertainty threshold, T_U. This threshold functions much like the omission threshold T_O, but is applied at the output rather than thresholding stage. Setting $T_U>T_G$ allows SEM to simulate the removal of uncertain responses from subjects' reports.

Intertrial interval

Between trials, the general context of all tokens is updated according to Equation 10-12, where $c=C_I+C_A$. The parameter C_I represents the number of episodes during the intertrial interval and the parameter C_A represents the number of contextual changes owing to attentional shifts during the intertrial interval (Chapter 5). The activations of phonological representations also decay according to Equation 10-13, with $c=C_I$.

Finally, the suppression of categorical and phonological representations of item u, $s_C^{(u)}$ and $s_P^{(u)}$, are reset to zero (with the assumption that the length of the intertrial interval and rate of decay of suppression make this a reasonable approximation).

Note that, in general, the real-time decay of suppression and activation might be uncoupled further from contextual change (Chapter 5) by parameterising presentation rates, length of retention interval, etc. (i.e., introducing new parameters in addition to C_P, C_D, C_R and C_I). This is beyond the scope of the present model.

Extension to Groups of Groups

SEM is readily extendible to any number of subgroupings of a sequence, by assuming that each boundary between groupings can be marked by start and end markers. With L levels of grouping, the positional codes are given by $\boldsymbol{p}_1, \boldsymbol{p}_2, \boldsymbol{p}_3 ... \boldsymbol{p}_L$, the general context by $\boldsymbol{p}_{L+1}$ and the strength with which token t is cued by the positional cue for response r is:

$$q^{(t)}(r) = \prod_{k=1}^{L+1} \overline{m}(\boldsymbol{p}_k^{(t)}, \boldsymbol{q}_k^{(r)}, d_k, M_k) \qquad \text{Equation 10-16}$$

The positional uncertainty functions for a sequence of $i,j=1..N_P$ positions coded by $k=1..L$ start and end markers is given by:

$$f = f_1 f_2 \ldots f_L \qquad f_k(i,j) = o(\boldsymbol{p}_k^{(i)}, \boldsymbol{p}_k^{(j)}) \qquad \text{Equation 10-17}$$

Implementational Details

The single-trial and multiple-trial versions of SEM have been written as computer programs in C to run on Unix. Both are available from the author on request (as is the program used to analyse the reports produced by SEM and by subjects), though the above formalism should be sufficient for one to implement their own version.

In addition to specifying parameter values, the single-trial program requires three further arguments, one representing the list, one representing the vocabulary and one representing the set of phonologically confusable items in the vocabulary (each item is represented by a single character). The program outputs two files, one with N_L copies of the specified list and one with the corresponding N_L reports. The multiple-trial program on the other hand reads the N_L lists from a file (often the same lists given to subjects), and outputs a file with one report of each list.

In theory, SEM does not assume a limit on the number of tokens in short-term memory. In practice however, only the most recent tokens can ever be retrieved, assuming continual context drift. The multiple-trial program therefore stores a finite number of tokens, specified by the parameter N_T, and functions as a FIFO stack in which the oldest token is overwritten by the newest. In practice, ensuring $N_T>4N_P$ tokens is sufficient to give reasonable levels of proactive interference.

Finally, both versions of SEM require an additional “random seed”, which determines the exact values selected from the model’s random generation function (algorithm AS 183 from Applied Statistics). This seed of course has negligible effect the asymptotic behaviour of the model, when N_L is large, but can produce different fits when N_L is small. In all fits herein, this seed was constant at *0*.

Summary of Fits

A table with the complete set of parameter values for each fit in Chapters 5 and 6 is given below, together with four additional fits illustrating further properties of SEM. Parameter values indicated with a hyphen are irrelevant to a fit (e.g., the value of P_S when no confusable items are specified); parameter values indicated with an asterix vary between simulations within a fit (e.g., to simulate different experimental conditions), and their values are given in the text above the tables.

Fit 1. Primacy, Recency, Locality and Fill-in

The single-trial version was fitted to the error position curve in the Long condition of Experiment 2 (one simulation). Setting $S_{0,I}=1.00$, $S_I=0.80$, this fit had three effective free parameters $E_{0,I}$, E_I and G_C. Remaining parameters were fixed at *0.00*.

Given the list *12345* and vocabulary *12345*, parameter values were:

N_P	N_T	N_V	N_G					N_L
5	*5*	*5*	*0*					10^5
$S_{0,I}$	S_I	$E_{0,I}$	E_I		$S_{0,G}$	S_G	$E_{0,G}$	E_G
1.00	*0.80*	*0.60*	*0.48*		-	-	-	-
D_I	D_G		M_I	M_G		G_C	G_P	G_G
0.00	-		*0.00*	-		*0.14*	*0.00*	*0.00*
T_O	T_G		P_S	P_D		A_P		R_S
0.00	*0.00*		-	-		*0.00*		*0.00*

Fit 2. Omissions

The single-trial version was fitted to transpositions and omissions (including intrusions) in the Long condition of Experiment 2 (one simulation). All parameters were fixed from Fit 1, except the new free parameter T_O.

Given the list *12345* and vocabulary *12345*, parameter values were:

N_P	N_T	N_V	N_G					N_L
5	*5*	*5*	*0*					10^5
$S_{0,I}$	S_I	$E_{0,I}$	E_I		$S_{0,G}$	S_G	$E_{0,G}$	E_G
1.00	*0.80*	*0.60*	*0.48*		-	-	-	-
D_I	D_G		M_I	M_G		G_C	G_P	G_G
0.00	-		*0.00*	-		*0.14*	*0.00*	*0.00*
T_O	T_G		P_S	P_D		A_P		R_S
0.48	*0.00*		-	-		*0.00*		*0.00*

Fit 3. Repetitions

The single-trial version was fitted to transpositions, omissions and repetitions in the PN condition of Experiment 1 (one simulation). All parameters were fixed from Fit 2, except N_P, N_T and N_V, for the longer lists, and G_C, T_O and R_S as the three free parameters.

Given the list *123456* and vocabulary *123456*, parameter values were:

N_P	N_T	N_V	N_G					N_L
6	*6*	*6*	*0*					10^5
$S_{0,I}$	S_I	$E_{0,I}$	E_I		$S_{0,G}$	S_G	$E_{0,G}$	E_G
1.00	*0.80*	*0.60*	*0.48*		-	-	-	-
D_I	D_G		M_I	M_G		G_C	G_P	G_G
0.00	-		*0.00*	-		*0.08*	*0.00*	*0.00*
T_O	T_G		P_S	P_D		A_P		R_S
0.32	*0.00*		-	-		*0.00*		*0.50*

Fit 4. Phonological Confusions

The single-trial version was fitted to transpositions, omissions and confusions in all four conditions of Experiment 1 (four simulations). Parameters were maintained from Fit 3, except the fixed parameters N_V=12, A_P=1.00, P_D=0.00 and three free parameters G_P, T_O, P_S.

Given the lists *RHKYMQ* (condition PN), *BMGQVK* (condition AC), *KGQVMB* (condition AN), and *VBGDPT* (condition PC), a vocabulary of *RHKYMQVBGDPT* and confusable set *VBGDPT,* parameter values were:

N_P	N_T	N_V	N_G					N_L
6	6	12	0					10^5
$S_{0,I}$	S_I	$E_{0,I}$	E_I		$S_{0,G}$	S_G	$E_{0,G}$	E_G
1.00	0.80	0.60	0.48		-	-	-	-
D_I	D_G		M_I	M_G		G_C	G_P	G_G
0.00	-		0.00	-		0.08	0.30	0.00
T_O	T_G		P_S	P_D		A_P		R_S
0.90	0.00		0.75	0.00		1.00		0.50

Fit 5. List Length, Grouping and Interpositions

The single-trial version was fitted to the conditions of Experiment 2 (four simulations). Parameters were maintained from Fit 4, except the fixed parameters $S_{0,G}$=1.00, S_G=0.80, N_P, N_T, N_G, N_I, and the eight free parameters $E_{0,I}$, E_I, $E_{0,G}$, E_G, D_I, M_I, D_G and M_G.

Parameters N_P, N_T, N_G, N_I, $E_{0,I}$, E_I and D_I varied between the ungrouped and grouped conditions. In the ungrouped conditions N_P, N_T, $N_I(1)$ were equal to the list length, and $N_G=1$, $N_I(2)=N_I(3)=0$, $E_{0,I}=0.60$, $E_I=0.60$, $D_I=0.04$. In the grouped condition, $N_P=N_T=9$, $N_G=3$, $N_I(1)=N_I(2)=N_I(3)=3$, $E_{0,I}=1.00$, $F_I=0.20$, $D_I=0.16$.

Given lists *1234567* (condition U7), *12345678* (condition U8), *123456789* (conditions U9 and G9), and a vocabulary of *0123456789*, parameter values were:

N_P	N_T	N_V	N_G	$N_I(1)$	$N_I(2)$	$N_I(3)$		N_L
*	*	*10*	*	*	*	*		10^5
$S_{0,I}$	S_I	$E_{0,I}$	E_I		$S_{0,G}$	S_G	$E_{0,G}$	E_G
1.00	*0.80*	*	*		*1.00*	*0.80*	*0.60*	*0.80*
D_I	D_G		M_I	M_G		G_C	G_P	G_G
*	*0.08*		*0.40*	*0.85*		*0.08*	*0.30*	*0.00*
T_O	T_G		P_S	P_D		A_P		R_S
0.90	*0.00*		*0.75*	*0.00*		*1.00*		*0.50*

Fit 6. Intertrial Interval and Protrusions

The multiple-trial version was fitted to both conditions of Experiment 3 (two simulations). All parameter values were maintained from Fit 5, except the new fixed parameters $E_{0,C}=1.00$, $C_P=C_R=1$, $C_D=3$, and the five free parameters E_C, R_P, C_A, G_C and T_O.

The parameter C_I varied between conditions (according to the experimental design). In the Long condition $C_I=20$; in the Short condition $C_I=2$.

Given 100,000 copies of the lists given to subjects, and a vocabulary of *YGVPWHCK MLSBFT* (representing words used; none of which were confusable), parameter values were:

N_P	N_T	N_V	N_G	$N_I(1)$			N_M	N_L
5	*20*	*14*	*1*	*5*			-	10^5
$S_{0,I}$	S_I	$E_{0,I}$	E_I		$S_{0,G}$	S_G	$E_{0,G}$	E_G
1.00	*0.80*	*0.60*	*0.60*		*1.00*	*0.80*	*0.60*	*0.80*
$E_{0,C}$	E_C			C_P	C_D	C_R	C_I	C_A
1.00	*0.98*			*1*	*3*	*1*	*	*20*
D_I	D_G		M_I	M_G		G_C	G_P	G_G
0.04	*0.08*		*0.40*	*0.85*		*0.10*	*0.30*	*0.00*
T_O	T_G	T_U	P_S	P_D		A_P	R_P	R_S
0.70	*0.00*	*0.00*	*0.75*	*0.00*		*1.00*	*0.05*	*0.50*

Fit 7. Interpositions in Variable Groups

The multiple-trial version was fitted to all three conditions of Experiment 4, with and without guesses (six simulations). Parameters were maintained from Fit 5, except the fixed parameters N_G, N_I, $G_G=0.30$, $T_O=0.00$, $C_P=C_R=1$, $C_D=C_I=0$, and the five free parameters D_G, M_G, G_C, T_G and T_U.

The parameters N_G, N_I, $E_{0,I}$ and E_I varied between conditions (according to experimental design and Fit 5). In the Ungrouped condition $N_G=1$, $N_I(1)=7$, $E_{0,I}=0.60$, $E_I=0.60$; in the Grouped 3-4 condition, $N_G=2$, $N_I(1)=3$, $N_I(2)=4$; $E_{0,I}=1.00$, $E_I=0.20$, and in the Grouped 4-3 condition, $N_G=2$, $N_I(1)=4$, $N_I(2)=3$, $E_{0,I}=1.00$, $E_I=0.20$. The parameter T_U varied to simulate the removal of guesses. with guesses, $T_U=0.00$; without, $T_U=1.10$.

Given 100,000 copies of the lists given to subjects, and a vocabulary of *GVCLBFT* (representing words used; none of which were confusable), parameter values were:

N_P	N_T	N_V	N_G	$N_I(1)$	$N_I(2)$		N_M	N_L
7	*28*	*7*	*	*	*		-	10^5
$S_{0,I}$	S_I	$E_{0,I}$	E_I		$S_{0,G}$	S_G	$E_{0,G}$	E_G
1.00	*0.80*	*	*		*1.00*	*0.80*	*0.60*	*0.80*
$E_{0,C}$	E_C			C_P	C_D	C_R	C_I	C_A
1.00	*0.98*			*1*	*0*	*1*	*0*	*20*
D_I	D_G		M_I	M_G		G_C	G_P	G_G
0.04	*0.10*		*0.40*	*0.95*		*0.06*	*0.30*	*0.30*
T_O	T_G	T_U	P_S	P_D		A_P	R_P	R_S
0.00	*0.90*	*	*0.75*	*0.00*		*1.00*	*0.05*	*0.50*

Fit 8. Protrusions in Variable Lists

The multiple-trial version was fitted to both conditions of Experiment 5, with and without guesses (four simulations). Parameters were maintained from Fit 7, except the one free parameter G_C.

No parameters changed across conditions, except for N_P which depended on the list length. Note that the value of N_M was only relevant to the Variable condition, were list lengths varied unpredictably. This was fixed as N_M=5, reflecting in the minimum expected list length in that condition. The parameter T_U varied to simulate the removal of guesses as in Fit 7: with guesses, $T_U=0.00$; without, $T_U=1.10$.

Given 100,000 copies of lists given to subjects, and a vocabulary of *YGVPWHCK MLSBFT* (representing words used; none of which were confusable), parameter values were:

N_P	N_T	N_V	N_G	$N_I(1)$			N_M	N_L
*	*24*	*14*	*1*	*			*5*	10^5
$S_{0,I}$	S_I	$E_{0,I}$	E_I		$S_{0,G}$	S_G	$E_{0,G}$	E_G
1.00	*0.80*	*0.60*	*0.60*		*1.00*	*0.80*	*0.60*	*0.80*
$E_{0,C}$	E_C			C_P	C_D	C_R	C_I	C_A
1.00	*0.98*			*1*	*0*	*1*	*0*	*20*
D_I	D_G		M_I	M_G		G_C	G_P	G_G
0.04	*0.10*		*0.40*	*0.95*		*0.01*	*0.30*	*0.30*
T_O	T_G	T_U	P_S	P_D		A_P	R_P	R_S
0.00	*0.90*	*	*0.75*	*0.00*		*1.00*	*0.05*	*0.50*

Additional Fits not Reported in Chapters 5 and 6

Fit 9. Retention Interval and Phonological Confusions

This was a qualitative fit of the multiple-trial version showing an interaction between phonological similarity and delay for each list-type in Experiment 1 (sixteen simulations). Parameters were fixed from Fit 8, except N_P=N_I=6, $N_V=12$, $T_U=0.00$ and $T_O=0.70$ (i.e., uncertain responses were included but omissions were added). The value of C_D varied from *0* to *5* to *10* to *20*, to simulate the length of a filled retention interval (in seconds).

Given 100,000 copies of the lists *RHKYMQ* (condition PN), *BMGQVK* (condition AC), *KGQVMB* (condition AN), *VBGDPT* (condition PC), a vocabulary *RHKYMQVBGDPT* and confusable set *VBGDPT*, parameter values were:

N_P	N_T	N_V	N_G	$N_I(1)$			N_M	N_L
6	24	24	1	6			-	10^5
$S_{0,I}$	S_I	$E_{0,I}$	E_I		$S_{0,G}$	S_G	$E_{0,G}$	E_G
1.00	0.80	0.60	0.60		1.00	0.80	0.60	0.80
$E_{0,C}$	E_C			C_P	C_D	C_R	C_I	C_A
1.00	0.98			1	*	1	0	20
D_I	D_G		M_I	M_G		G_C	G_P	G_G
0.04	0.10		0.40	0.95		0.01	0.30	0.30
T_O	T_G	T_U	P_S	P_D		A_P	R_P	R_S
0.70	0.90	0.00	0.75	0.00		1.00	0.05	0.50

The sawteeth shape of error position curves for alternating lists disappeared as the retention interval increased (upper panel of Figure A-1). In other words, the phonological similarity effect disappeared and confusions fell to chance levels. Indeed, when $C_D=20$, performance on PC lists was almost identical to PN lists. This arises because the phonological activations decay to zero as the delay increases. This is consistent with previous results, but has not been demonstrated empirically in the striking fashion shown here with alternating lists.

The retention interval also affected the pattern of errors. As the phonological activations decayed, the incidence of omissions and intrusions increased, faster than that of transpositions (lower panel of Figure A-1). No study has shown this specific pattern: The increase in omissions and intrusions (and decrease in confusions) are predictions of SEM.

Unlike the single-trial version of SEM, the feedback of responses in the multiple-trial version means that errors on confusable items do impair recall of subsequent nonconfusable items to a small extent (there were small differences of about 5% between nonconfusables in alternating and nonconfusable curves). This is not necessarily a problem, given that such a small effect was suggested by the third meta-analysis of Chapter 4.

Finally, the proportion of intrusions that were protrusions decreased as retention interval increases, reflecting the increasing effects of guessing (see Fit 6 in Chapter 5).

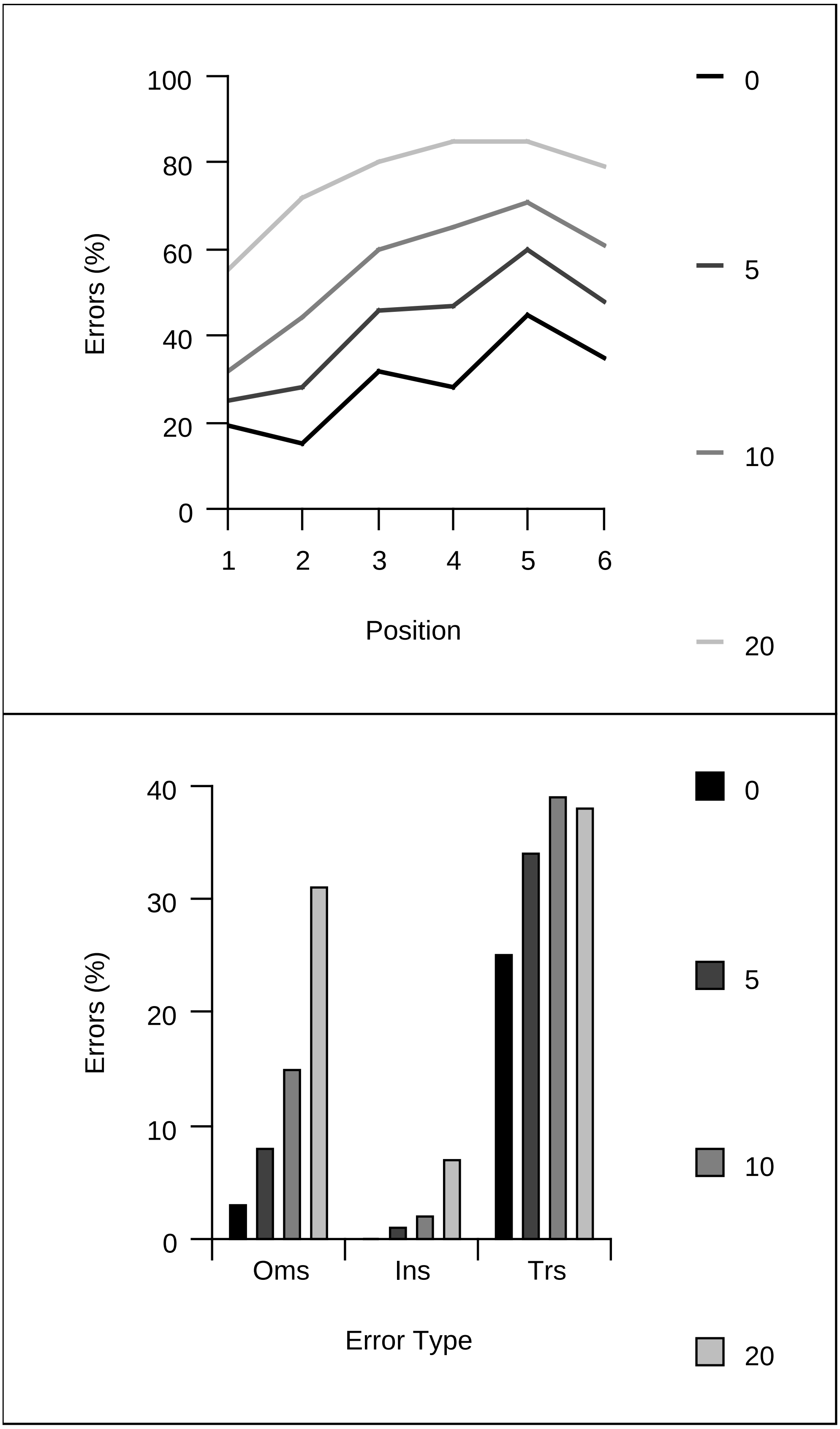

Figure A-1: Effects of retention interval on error position curves (upper panel) and error types (lower panel) for condition AC in Fit 9.
(Numbers refer to parameter C_D; Oms=omissions, Ins=intrusions and Trs=transpositions.)

Fit 10. List Length and Span

This was a qualitative fit of the multiple-trial version to list length effects (eight simulations). Parameters were fixed from Fit 9, except $C_D=0$, $G_C=0.06$, $N_T=36$, $N_T=10$, and N_P, which increased from 2-9, reflecting increases in list length.

Given 10,000 copies of lists drawn from *0123456789*, parameter values were:

N_P	N_T	N_V	N_G	$N_I(1)$			N_M	N_L
*	*36*	*10*	*1*	*			-	10^4
$S_{0,I}$	S_I	$E_{0,I}$	E_I		$S_{0,G}$	S_G	$E_{0,G}$	E_G
1.00	*0.80*	*0.60*	*0.60*		*1.00*	*0.80*	*0.60*	*0.80*
$E_{0,C}$	E_C			C_P	C_D	C_R	C_I	C_A
1.00	*0.98*			*1*	*0*	*1*	*0*	*20*
D_I	D_G		M_I	M_G		G_C	G_P	G_G
0.04	*0.10*		*0.40*	*0.95*		*0.06*	*0.30*	*0.30*
T_O	T_G	T_U	P_S	P_D		A_P	R_P	R_S
0.70	*0.90*	*0.00*	*0.75*	*0.00*		*1.00*	*0.05*	*0.50*

Longer lists increased errors on all positions, including the first (upper panel of Figure A-2). Note that error position curves for lists of more than seven items may not resemble those found empirically, because of the tendency for subjects to spontaneously group such lists (Experiment 2). Importantly, SEM produced the characteristic inverse-S shaped curves of lists correct against list length (lower panel of Figure A-2).

Fit 11. Word Length and Articulation Rate

This was a qualitative fit of the multiple-trial versions to effects of word-length and articulation rate (forty simulations). Parameters were fixed from Fit 9, except for a factorial combination of $C_P=C_R=1...5$, reflecting word-length, and $N_P=2...9$, reflecting list length. The values of C_P and C_R were greater for long words than short words, with the assumption that they allow a greater opportunity for decay during presentation (i.e., ignoring rehearsal during presentation for simplicity; Chapter 5).

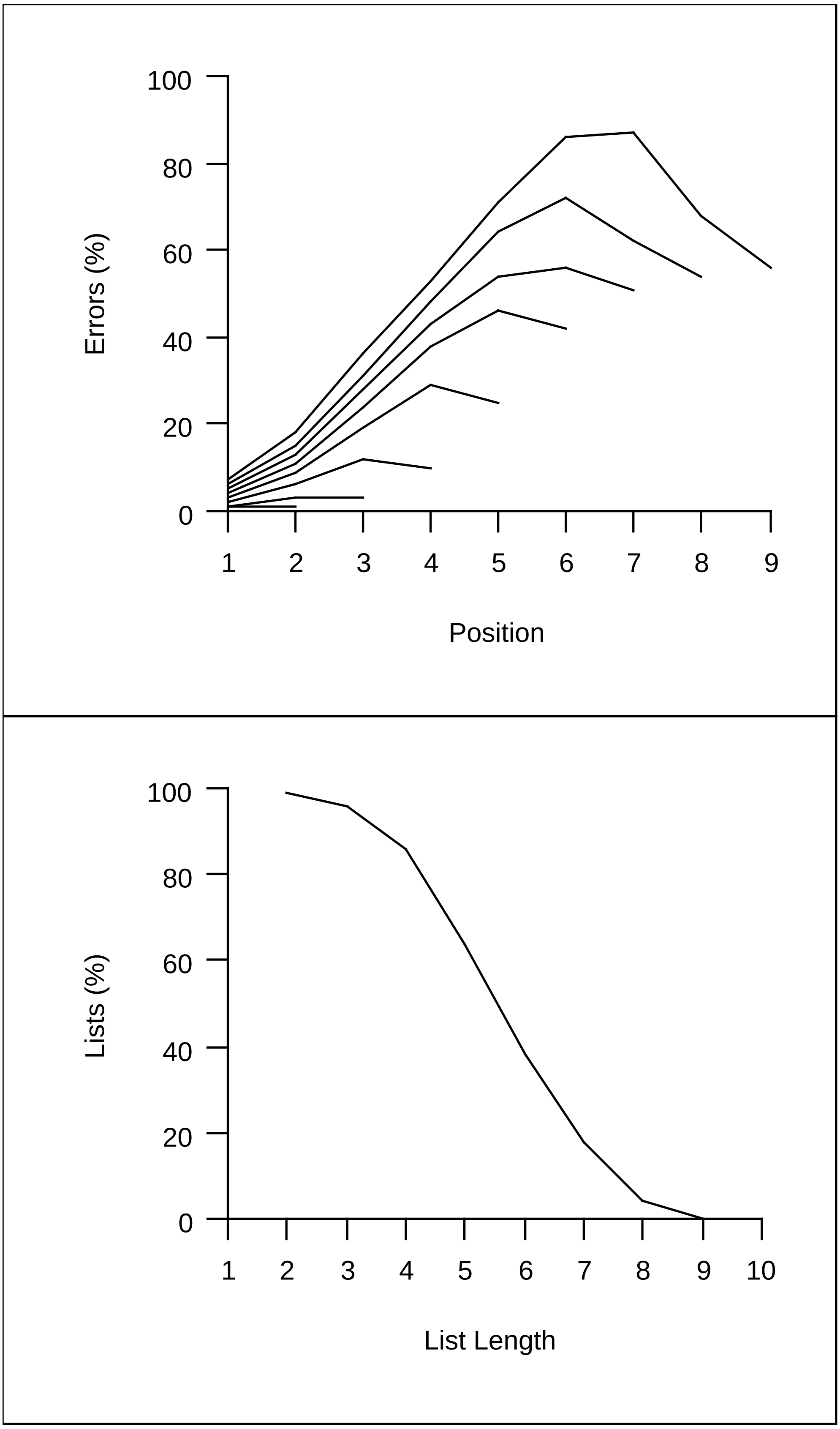

Figure A-2: Effect of list length on error position curves (upper panel) and lists correct (lower panel) in Fit 10.

Given 10,000 copies of lists drawn from *0123456789*, parameter values were:

N_P	N_T	N_V	N_G	$N_I(1)$			N_M	N_L
*	*36*	*10*	*1*	*			-	10^4
$S_{0,I}$	S_I	$E_{0,I}$	E_I		$S_{0,G}$	S_G	$E_{0,G}$	E_G
1.00	*0.80*	*0.60*	*0.60*		*1.00*	*0.80*	*0.60*	*0.80*
$E_{0,C}$	E_C			C_P	C_D	C_R	C_I	C_A
1.00	*0.98*			*	*0*	*	*0*	*20*
D_I	D_G		M_I	M_G		G_C	G_P	G_G
0.04	*0.10*		*0.40*	*0.95*		*0.06*	*0.30*	*0.30*
T_O	T_G	T_U	P_S	P_D		A_P	R_P	R_S
0.70	*0.90*	*0.00*	*0.75*	*0.00*		*1.00*	*0.05*	*0.50*

Longer words decreased spans (the 50% correct level in upper panel of Figure A-3). Closer analysis of errors showed that longer words increased both transpositions and omissions, particularly towards the end of recall, in agreement with unpublished data (Page & Norris, 1996a).

The parameters $C_P=C_R=C$ were assumed to be related to the number of syllables in words, W, by the formula $W=(C+1)/2$ (i.e., an extra syllable corresponded to an increase of 2 in C_P and C_R). They were also converted into speeded articulation rate, R, by the formula $R=3.1-0.6C$. The latter formula gives a good approximation to the rates determined by Page & Norris (1996b) in their fit to Hulme et al (1991). The resulting relationship between span and articulation rate is shown in the lower panel of Figure A-3. The relationship is near-linear ($R^2=.93$), with an approximate slope of *0.88* and intercept of *3.15*, in reasonable agreement with the data of Hulme et al (1991). The relationship between span and rate departs most from linearity for the slowest and fastest rates. It is noteworthy that these rates are at or beyond the limits normally achieved experimentally, and thus the quadratic component of the span-rate curve reflects a testable prediction of SEM (though other relationships between span and rate may be possible with different parameter values).

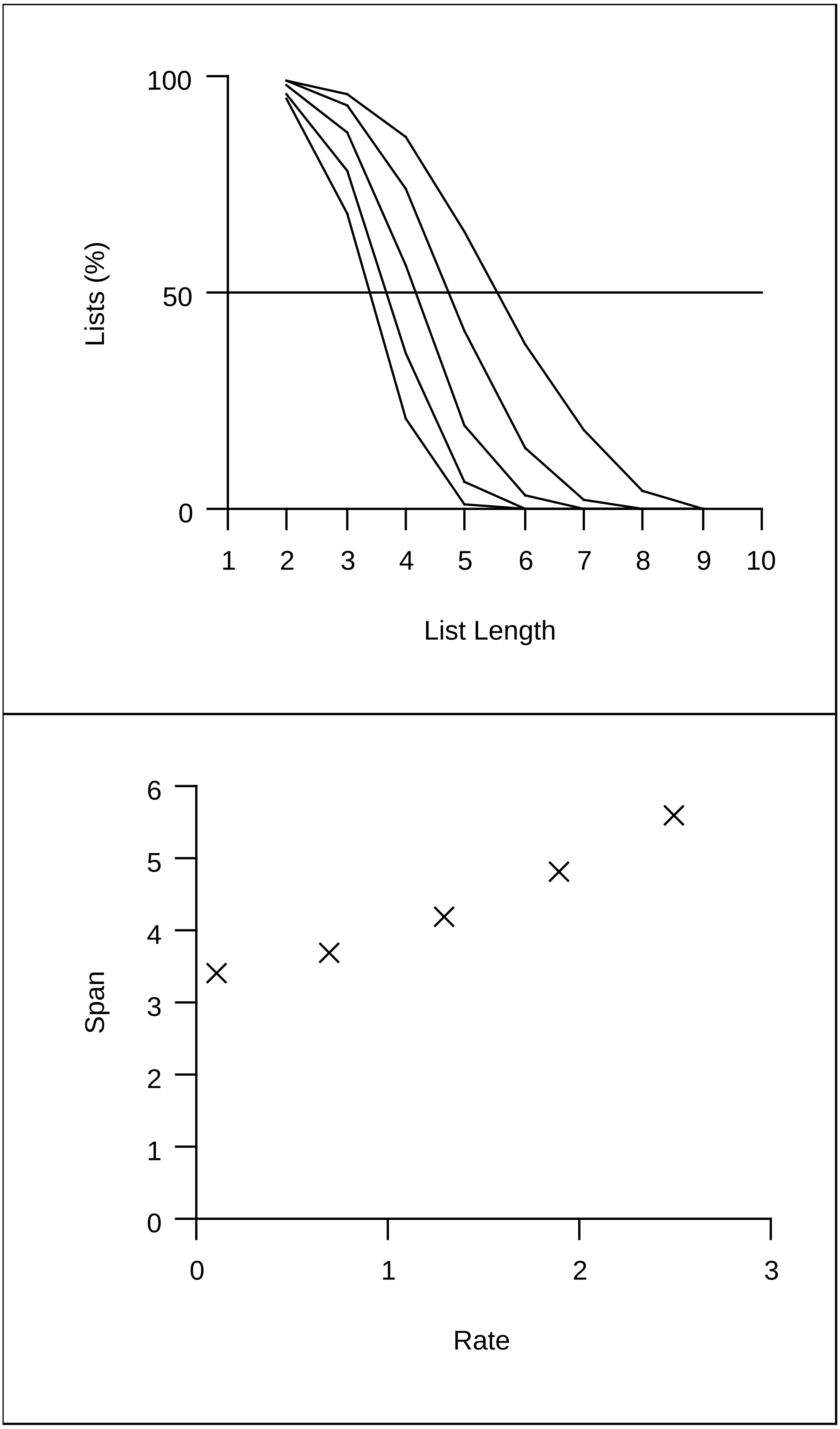

Figure A-3: Effect of word-length on lists correct (upper panel; curves to the left represent longer words) and effect of articulation rate on span (lower panel) in Fit 11.

Fit 12. Modality and Suffix Effects

This was a qualitative fit of the multiple-trial version to modality and suffix effects in recall of eight items (three simulations). Parameters were fixed from Fit 11, except for the values $N_P=8$, $N_T=32$, $E_I=0.40$ and value of $E_{0,I}$, which changed between conditions.

The parameter $E_{0,I}=0.60$ for auditory lists and $E_{0,I}=0.20$ for visual lists. The latter reflected the assumption that the end marker for auditory lists is stronger than for visual lists. The value $E_{0,I}=0.24\ (=0.60.E_I)$ for auditory lists with a suffix reflected the assumption that the suffix was marked in the last position, rather than the last item (i.e., the suffix was unavoidably grouped together with the list items, so that position was coded as if there were nine positions, though only the eight list items competed for recall). This fit did not take into account the additional delay caused by a suffix, though this could be modelled simply by increasing C_R, which would affect mainly middle items (Baddeley & Hull, 1979).

Given 100,000 copies of lists drawn from *0123456789*, parameter values were:

N_P	N_T	N_V	N_G	$N_I(1)$			N_M	N_L
(8)	*32*	*10*	*1*	*8*			-	10^5
$S_{0,I}$	S_I	$E_{0,I}$	E_I		$S_{0,G}$	S_G	$E_{0,G}$	E_G
1.00	*0.80*	*	*0.40*		*1.00*	*0.80*	*0.60*	*0.80*
$E_{0,C}$	E_C			C_P	C_D	C_R	C_I	C_A
1.00	*0.98*			*1*	*0*	*1*	*0*	*20*
D_I	D_G		M_I	M_G		G_C	G_P	G_G
0.04	*0.10*		*0.40*	*0.95*		*0.06*	*0.30*	*0.30*
T_O	T_G	T_U	P_S	P_D		A_P	R_P	R_S
0.70	*0.90*	*0.00*	*0.75*	*0.00*		*1.00*	*0.05*	*0.50*

The modality advantage for auditory presentation extended over the last two or three positions, but was removed by an additional suffix (Figure A-4). The auditory advantage reflected a decrease in both omissions and transpositions, in agreement with unpublished data (Page & Norris, 1996a). Note that the exact values of $E_{0,I}$ and E_I needed to produce this pattern do depend on the values of other parameters (such as the noise G_C). The purpose of this fit is simply to show how SEM's assumption of marking the end of a list can in principle reproduce modality and suffix effects; further work is needed on how exactly auditory presentation or additional suffixes affect such marking.

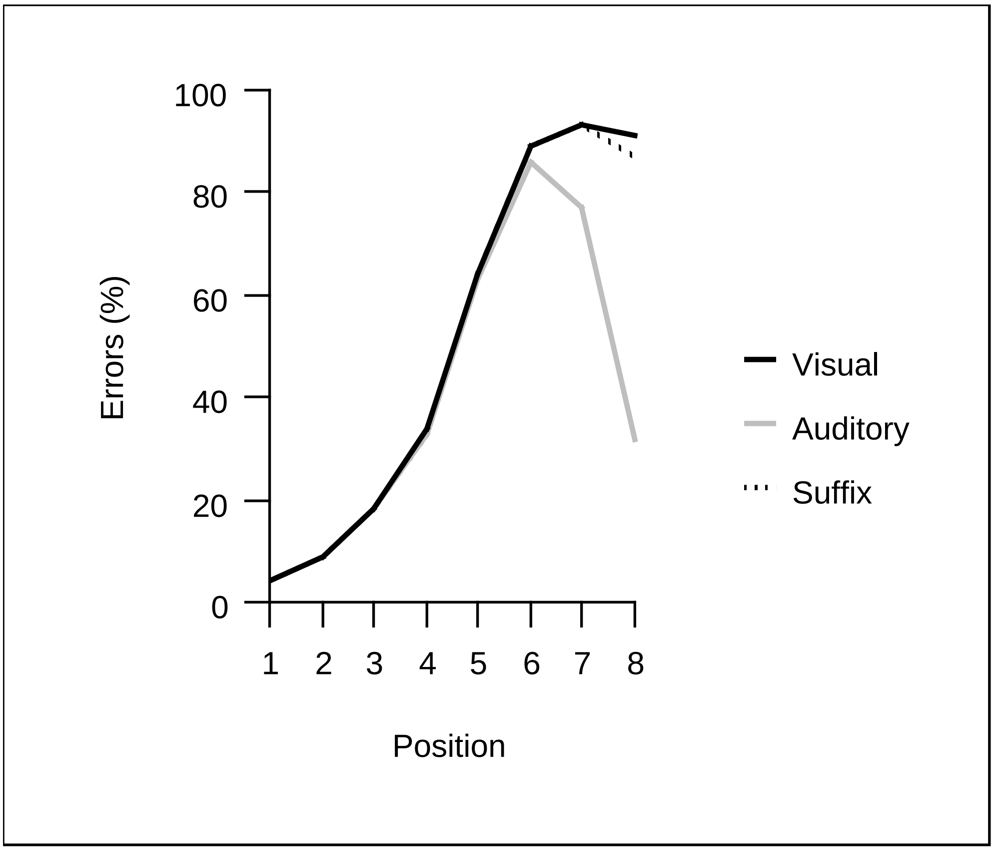

Figure A-4: Modality and suffix effects in Fit 12.

Glossary of Terms in SEM

Parameters

N_T	Number of tokens in short-term memory
N_V	Number of items in vocabulary
N_L	Number of lists
$N_P(l)$	Number of positions in list *l* (list length)
$N_G(l)$	Number of groups in list *l*
$N_I(g)$	Number of items in group *g* (group size)
N_M	Minimum expected list length
$S_{0,I}$	Initial value of start marker for item position in group
S_I	Decay rate of start marker for item position in group
$E_{0,I}$	Initial value of end marker for item position in group
E_I	Decay rate of end marker for item position in group
$S_{0,G}$	Initial value of start marker for group position in list
S_G	Rate of change of start marker for group position in list
$E_{0,G}$	Initial value of end marker for group position in list
E_G	Rate of change of end marker for group position in list
$E_{0,C}$	Value for current general context
E_C	Rate of change of general context
D_I	SD of Gaussian noise in item position codes
D_G	SD of Gaussian noise in group position codes

M_I	Positional Match Criterion for item position codes
M_G	Positional Match Criterion for group position codes
G_C	SD of Gaussian noise in categorical competition
G_P	SD of Gaussian noise in phonological competition
G_G	SD of Gaussian noise in guessing
T_O	Omission threshold
T_G	Guessing threshold
T_U	Uncertainty threshold
P_S	Phonological similarity between similar items
P_D	Phonological similarity between dissimilar items
A_P	Baseline Activation of phonological representations
R_P	Rate with which phonological activation decays
R_S	Rate with which suppression decays
C_P	Effective presentation rate (ignoring rehearsal)
C_D	Length of filled delay during retention interval
C_R	Effective recall rate (ignoring rehearsal)
C_I	Length of intertrial interval
C_A	Contextual/Attentional shift during intertrial interval

Indices

$l=1..N_L$	List number
$p=1..N_P(l)$	Item (input) position in list *l*
$r=1..N_P(l)$	Response (output) position in recall of list *l*

$i=1..N_I(g)$	Item position in group *g*
$g=1..N_G(l)$	Group position in list *l*
$c=0..inf$	Number of episodes for contextual change/decay
$u=1..N_V$	Index for categorical representation of item
$v=1..N_V$	Index for phonological representation of item

Variables

$x(i)$	Strength of start marker at position *i*
$y(i)$	Strength of end marker at position *i*
$\boldsymbol{p}_I^{(t)}$	Positional code for item position in group for token *t*
$\boldsymbol{p}_G^{(t)}$	Positional code for group position in list for token *t*
$\boldsymbol{p}_C^{(t)}$	Positional code for general context for token *t*
$\overline{m}(\boldsymbol{p},\boldsymbol{q})$	Thresholded match between positional codes ***p*** and ***q***
$m(\boldsymbol{p},\boldsymbol{q})$	Noisy match between positional codes ***p*** and ***q***
$o(\boldsymbol{p},\boldsymbol{q})$	Overlap between positional codes ***p*** and ***q***
$f(i,j)$	Positional uncertainty functions over positions *i,j*
$q^{(t)}(r)$	Cued strength of token t for response r
$c^{(u)}$	Competition strength of item *u*
$a^{(u)}$	Activation of item *u*
$s^{(u)}$	Suppression of item *u*
d	Noise in positional code
n	Noise in competition
$p(u,v)$	Phonological similarity between item u and item v