Advanced topics in RSA

Nikolaus Kriegeskorte
MRC Cognition and Brain Sciences Unit
Cambridge, UK
Advanced topics menu

• How does the *noise ceiling* work?

• Why is *Kendall’s tau a* often needed to compare RDMs?

• How can we *weight model features* to explain brain RDMs?

• What’s the difference between *dissimilarity, distance, and metric*?
How does the *noise ceiling* work?
Correlation distance \( \propto \) Euclidean distance\(^2 \)
(for normalised patterns)

\[
d^2 = (1 - r)^2 + (1 - r^2) \\
= 1 - 2r + r^2 + 1 - r^2 \\
= 2(1 - r)
\]
The accuracy of human IT dissimilarity matrix prediction, [group-average rank correlation], is shown in the graph. The highest accuracy any model can achieve is compared to other subjects’ average as the model. The accuracy above chance, p<0.001, is indicated with asterisks. The figure includes convolutional, fully connected, SVM discriminants, and a weighted combination of fully connected layers and SVM discriminants. The authors reference Khaligh-Razavi & Kriegeskorte (2014) and Nili et al. 2014 (RSA Toolbox).
The group-mean RDM minimises the sum of squared Euclidean distances to single-subject RDMs.

For rank RDMs, the group-mean RDM therefore minimises the sum of correlation distances (thus maximising the average correlation).

The average correlation to the group-mean of rank RDMs, thus provides a precise and hard upper bound on the average Spearman correlation any model can achieve.

For Kendall's tau a, an iterative procedure is needed to find the hard upper bound.
Estimating the noise ceiling

Nili et al. 2014
Why is Kendall’s tau a often needed to compare RDMs?

Kendall’s tau a is needed when testing categorical models that predict tied dissimilarities.
General correlation coefficient

\[ \Gamma = \frac{\sum a_{ij} \cdot b_{ij}}{\sqrt{\sum a_{ij}^2 \cdot \sum b_{ij}^2}} \]

**Pearson**
\[
a_{ij} = x_j - x_i \\
b_{ij} = y_j - y_i
\]

**Spearman**
\[
a_{ij} = \text{rank}(x_j) - \text{rank}(x_i) \\
b_{ij} = \text{rank}(y_j) - \text{rank}(y_i)
\]

**Kendall** (\(\tau_a\))
\[
a_{ij} = \text{sign}(x_j - x_i) \\
b_{ij} = \text{sign}(y_j - y_i)
\]

Kendall 1944
True model can lose to simplified step model
(according to most correlation coefficients)

Nili et al. 2014
True model can lose to simplified step model
(according to most correlation coefficients)

Nili et al. 2014

true model

categorical models (predicting tied ranks)
True model can lose to simplified step model
(according to most correlation coefficients)

Simulated data

Real data

Nili et al. 2014

true model
categorical models (predicting tied ranks)
True model can lose to simplified step model
(according to most correlation coefficients)
True model can lose to simplified step model (according to most correlation coefficients)
Kendall’s tau a chooses the true model over a simplified model (which predicts ties in hard cases) more frequently than Pearson r, Spearman r, tau b and tau c.
Conclusions

• Rank correlation can be useful for comparing RDMs when measurement errors might distort a simple linear relationship.

• When categorical models (predicting tied ranks) are used, Kendall’s tau a is an appropriate rank correlation coefficient.

• Kendall’s tau b & c, Spearman correlation, and even Pearson correlation all prefer models that predict ties for difficult comparisons to the true model.
How can we weight model features to explain brain RDMs?
accuracy of human IT dissimilarity matrix prediction
[group-average of Kendall’s $\tau_w$]

* accuracy above chance
p<0.001
(subjects and stimuli as fixed effects)

- layer 1, layer 2, layer 3, layer 4, layer 5, layer 6, layer 7, face, anim,
  all categories
- convolutional, fully connected, SVM discriminants

highest accuracy any model can achieve
other subjects’ average as model

SE (stimulus bootstrap)

Khaligh-Razavi & Kriegeskorte (2014), Nili et al. 2014 (RSA Toolbox)
Representational feature weighting with non-negative least-squares

\[ w_1 f_1 \quad w_2 f_2 \quad \ldots \quad \ldots \quad w_k f_k \]

\[ f_1 \quad f_2 \quad \ldots \quad \ldots \quad f_k \]

model RDM

weighted-model RDM

Khaligh-Razavi & Kriegeskorte (2014)
Representational feature weighting with non-negative least-squares

The squared distance RDM of weighted model features equals a weighted sum of single-feature RDMs.

\[
\hat{d}_{i,j}^2 = \sum_{k=1}^{n} \left[w_k f_k(i) - w_k f_k(j)\right]^2
\]

\[
= \sum_{k=1}^{n} w_k^2 \left[f_k(i) - f_k(j)\right]^2
\]

\[
w = \arg \min_{w \in \mathbb{R}^+} \sum_{i \neq j} \left[d_{i,j}^2 - \hat{d}_{i,j}^2\right]^2 = \arg \min_{w \in \mathbb{R}^+} \sum_{i \neq j} \left[d^2 - \sum_{k=1}^{n} w_k^2 \cdot \text{RDM}_k\right]_{i,j}^2
\]

\[w_k\] weight given to model feature \(k\)

\[f_k(i)\] model feature \(k\) for stimulus \(i\)

\[d_{i,j}\] distance between stimuli \(i,j\)

\(w\) is the weight vector \([w_1, w_2, ..., w_k]\) minimising the squared errors

Khaligh-Razavi & Kriegeskorte (2014)
What’s the difference between \textit{dissimilarity}, \textit{distance}, and \textit{metric}?
Dissimilarity measures

LD-t

Distances

correlation distance
squared Euclidean distance
Minkowski distance with p < 1

Metrics

d(x,y) = 0 ⇔ x = y

Euclidean distance
Mahalanobis distance
Minkowski distance with p ≥ 1